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About Mathematical Thinking in political Science

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تهدف هذه السلسلة الى اقامه الحوار العلمى بين المشتغلين بالبحث والتدريس فى علم السياسة وذلك لنشر بحوث تتناول وجهات نظر منهجيه أو دراسات تتعلق بالظواهر السياسية موضع الاهتمام الاكاديمى وتخضع بحوث السلسلة للتحكيم من قبل اثنين من الاساتذه المتخصصين ·

ملخص

تبدأ هذه الورقة بعرض نشأة فكرة " التآخى " بين عالم الطبيعة وعالم الانسان فى الفكر الميثولوجى الاغريقى من خلال قصة " ليتو " وطفليها " آرتميس " و " آبوللو " وما تقترحه هذه القصة من التداخل المعرفى بين العلوم الطبيعية و العلوم الانسانية ، وهى الفكرة التى تطورت فيما بعد الى ماسمى بالفيزياء الاجتماعية التى تقوم على امكانية دراسة السلوك الانسانى و الاجتماعى من خلال القوانين الفيزيائية ، وتعرض الورقة رأى المدرسة التى تؤيد هذا الاتجاه ، ورأى المدرسة المتحفظة التى تحذر من المغالاة فى تطبيق هذه الفكرة .

وتتطرق الورقة الى التميز بين التفكير الرياضى و الطرق الرياضية من حيث أن الثانية تمثل حقلا معرفيا قائما بذاته فى حين أن الاول يمثل منهجا يصلح للتطبيق فى الرياضيات كما فى غيرها من العلوم ، وتوضح الورقة ضرورة التخلص من القصور الذاتى الذى اصاب تطبيق الرياضيات فى العلوم الاجتماعية و الانسانية بداء الفيثاغورسية التى تتعامل مع الرياضيات على أنها علم دراسة الأرقام و الأشكال الهندسية ، وتؤكد على أن الرياضيات الحديثة لا تتعامل بالضرورة مع الارقام و الأشكال الهندسية وانما مع الجبريات التى تمثل بيئات فكرية مختلفة التكوين تخضع كل منها لمجموعة من المسلمات التى لا تتفق بالضرورة مع تلك التى تحكم الأرقام و الأشكال الهندسية .

كذلك تتطرق الورقة الى ضرورة التفكير الرياضى كمنهج للعلوم التى تبحث عن الحقيقة بعيدا عن القصور الذى يصاحب المنهج غير الرياضى الذى عادة ما تأخذ به العلوم السياسية مع عرض لبعض أوجه هذا القصور ، وتتعرض الورقة بشئ من التفصيل لفكرة التجريد من حيث هو ضرورة منهج ، مع استخدام فكرة التحليل النظمى فى فهم درجات و مراحل التجريد المختلفة ، وعرض العلاقة التلازمية بين التجريد من ناحية و الرياضيات الرمزية من ناحية أخرى .

تتضمن الورقة كذلك جزءا خاصا لعرض التجربة الفيثاغورسية فى تطبيق الرياضيات فى دراسة النظم السلوكية و بالذات فى بناء النظامين الدينى و السياسى ، مع عرض للأزمة الفلسفية التى صاحبت اكتشاف الفيثاغورسيين أنفسهم للأرقام المتسامية مع ما صاحب هذا الاكتشاف من انقسام الفيثاغورسيين الى رياضيين و سمعيين ، واتصال الرياضيين تحت قيادة آرضيتاس بأكاديمية أفلاطون ، و أثر ذلك على تطور وارتقاء الفكر الانسانى فى مجال العلوم عموما والعلوم السياسية على وجه الخصوص ، مع عرض للدروس المستفادة من هذه التجربة الفيثاغورسية فى مجال تطبيق الرياضيات واستخدامها ٠

أخيرا تنتهى الورقة بعرض لبعض الآراء المناهضة لتحقيق حلم ديكارت بفهم متكامل للكون تلعب فيه الرياضيات دور الريادة ، مع عرض لأوجه القوة و القصور فى هذه الآراء ٠

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There are two classes of persons: one class of those who will agree with you and will take your words as a revelation; another class to whom they will be uttterly unmeaning, and who will naturally deem them to be idle tales, for they see no sort of profit which is to be obtained from them. And therefore you had better decide at once with which of the two you are proposing to argue. You will very likely say with neither, and that your chief aim in carrying on the argument is your own improvement.

Plato

(Republic : Book VII)

PROLOGUE

Following Greek mythology [6], Coius: the original intelligence, and Phoebe: the original light that pierced darkness, had a daughter: the original truth Leto. At that time before the Time, Zeus, the governor of the world, had a wife: his twin sister Hera, source of protection and, consequently, of never–ending jealousy. She wanted Zeus to be hers, and only hers, but the guarantor of the universal order could not be the prisoner of his wife's love or, to be precise, her passions and egotistic pride. He had to *communicate* with all the *female* principles of the existence, and to have innumerable children to rule the universe. One of his renowned infidelities to Hera was his association with the poor Leto who, once pregnant, became the object of Hera's jealousy. She was banished and, after Hera's orders, tracked by the redoubtable serpent Python.

Simply because she did not please to anyone, after that Hera ordained that no land then under the sun give her any refuge, Leto wandered all around the world, finding no place nor people to accept her, until that Zeus intervened to end her errantry which risked to continue for ever. An island called Delos emerged from the sea, and on it Leto gave birth to a daughter, Artemis, who, soon after being born, delivered Leto of her second child, Apollo, *whose birth was an agony for the mother*.

Yet a child, Artemis asked her father, the great Zeus, a double–favour: to accord her an eternal virginity, and to give her the means for she can live in freedom with animals in the wild forests and mountains. Zeus was generous with his daughter, and Artemis became the godess, that is, the truth of the wild order.

As for Apollo, the twin brother of Artemis, he left the island of Delos, where he was born, and went to the Mount Parnassus where lived the serpent Python. He avenged his mother, then took possession of the city of Delphi where he erected his sanctuary, and became the god, that is, the truth of the human order. God of medecin, arts, and wisdom, he orders man's body and soul.

The two children of Leto lived each in his world, but they have never forgotten that they come from the same womb.

This beautiful story of the Greek mythology have been chosen as a penetration gate to our paper for simple reasons, some of which will be discussed later. The instant reason is to *suggest* that human and wild orders are simultaneously dependent and independent. They are dependent by the primitive truth giving them birth; but are independent by the way each of them expresses itself. For a profane, watching the external appearance of things, nothing would reveal the unicity of these two worlds. But for a scientific investigator, searching the remote causes of behaviour, the two worlds are twin brothers emanating from the same mother laws.

SOCIAL PHYSICS

In their paper entiteled *An Economic Derivation of the Gravity Law of Spatial Interaction,* J. H. Niedercorn and R. V. Bechdolt [12] wrote: *The fundamental idea underlying spatial interaction models is that the degree of interaction between two geographic areas is a function of (1) the degree of concentration of persons or things in the two areas and (2) a measure of the distance separating the two areas. This fundamental idea is analogous to, and was originally derived from, the Newtonian laws of force and energy. Gravity and potential models evolved as a part of the early work of social physicists who believed that social phenomena could be explained by physical laws.* One of the most enthusiast promoters of the so-called *Social Physics,* the astrophysicist John Quincy Stewart, wrote: *Social Physics can attain wide influence only if its supporters remember that physics too is one of the humanities. Indeed the term Law of Nature is common to natural science and jurisprudence. Social physics can be described as the repayment by physics to politics of the borrowed law, with the rich interest earned since the year 1609 when Bacon published The Advancement Of Learning.* [24].

In following a mathematical path, starting at elementary truths that we call axioms, man was able to reach transcendental truths of the wild nature, far from superstitious prejudices, subjective judgements, and hazardous speculations. In human and social sciences, man did not achieve the same performance, either because he did not follow the same mathematical path, or simply because mathematics was roughly

applied with its barbarian techniques fitted to deal with differently postulated problems. In the conclusion of their paper, Niedercorn and Bechdolt wrote: *The so-called Gravity Law of spatial interaction can be logically derived from the economic principle of utility maximization, rather than from the vague and irrelevant concepts of social physics.* [12].

In the sanctuary of Apollo, two maximes had the supremacy: the first is *Know Thyself*, the other is *Nothing in Excess*. Apollo and Artemis have the same *mother*, but not the same *body*. Ancient Greeks recognized this fact; we have not to forget it.

MATHEMATICAL THINKING

This paper is about *mathematical thinking*, not about *mathematical techniques*. Mathematical techniques constitute a subject matter and self-contained knowledge field, while mathematical thinking is rather an approach to tackle problems in mathematics and in other fields as well. In this approach, nothing is sacred but the truth; and truth could not be reached if fallacies are introduced by force, or by habits and traditions, to decide of what is allowed to be true and what is not. Our objective is to see our world as it *truly* is, not as we think or believe it to be, nor as it would please us, or *them*, if it had been.

In mathematics, as in sciences that apply it correctly, transcendental truths, that is, truths that are not self-evident, are not admitted unless proven. In political science, such truths are usually admitted through discourses that may, and often do, persuade men in flirting their passions, not in awaking their reason. Whereas in perfect sciences we are concerned with the objects of thought, political science has the imperfection to be concerned, to one degree or another, with the thinking subject. Blaise PASCAL, the French mathematician and philosopher, noticed that *opinions are admitted into the soul through two entrances,... understanding and will. The most natural entrance*, following his judgement, *is the understanding, for we should never agree to anything but demonstrated truths, but the more usual entrance, though against nature, is the will, for all men whatsoever are almost always led into belief not because a thing is proved but because it is pleasing.*

Following PASCAL, *this way is low, unworthy, and foreign to our nature... Each of us professes to give his belief and even his love only where he knows it is deserved* [14].

PASCAL, however, emphasized the critical case where *things we wish to persuade of are firmly based on known truths, but at the same time are opposed to the pleasures which touch us most nearly*. He expressed his fear of the great danger that arises when the *imperious soul, whose boast was to act only by reason, follows by a rash and shameful choice the desires of a corrupt will, whatever resistance the too enlightened mind may offer* [14]. As we shall see later in this paper, PASCAL himself was a victim of this dangerous trap. In a such situation, he thought, *truth and pleasure hang doubtfully in the balance, and the knowledge of one and the feeling of the other engage in a struggle whose outcome is most uncertain since, to judge of it, we should have to know all that takes place in the innermost part of a man, which the man himself almost never knows* [14]. A such struggle between truth and pleasure can always come to its conclusion if mathematics is admitted as an unbiased balance and honest arbitrator. Unbiasedness here is to be understood as being between our soul and the object of thought since between truth and pleasure, mathematics is always biased to the former.

Some of the ideas expressed in this paper might provoke the reader, since touching his (or her) hereditary beliefs. This is intentional! By experience, I have found that students of human and social sciences, in general, and those of political science, in particular, averse mathematics, not because of mathematics itself, but because they are not trained to mathematical thinking. Verbal reasoning to which they are accustomed, allowed them to reach conclusions without great effort. Of course, I do suppose that mental brutality is not an effort, but a mere idleness of mind. Their argumentation looks like a path through the jungle: if a tree obstructs their way, they remove it roughly, or jump over *cleverly*. No routes are traced, and no maps are drawn. Travelling in all directions, they cut down more and more trees; and the jungle, deprived of its landmarks, becomes a desert with its mirages that might deceive them before that they deceive others. This becomes later one of

their most important handicaps when they try to move from the phase of *study* to the phase of *research.* This is not, of course, a universal truth to be generalized but, rather, an observation that should be mentioned and critically put under investigation. Once more, I am not talking about mathematical techniques but, rather, about mathematical thinking. In the faculty of economics and political science at Cairo university, for instance, we can find many examples of research activities in political science that belong to the mathematical school though applying absolutely no mathematical techniques.

MATHEMATICAL TECHNIQUES

Mathematical Thinking for Political Science?! What a lingo!, would murmur deridingly some students, and even some professors, of human and social sciences. For them, mathematics is *quantitative* by definition, while human and social sciences are *qualitative* by nature since they deal with the individual and collective behaviour of man. Behaviour, they would argue, is neither measurable nor deterministic. How mathematics could deal with variables that are not subject to measure and, moreover, do not obey the determinism of mathematical equations?!

To introduce mathematics into political science, we have to quit the Pythagorean cage through the bars of which mathematics is seen as *shapes* and *numbers.* In *modern* mathematics, that some peoples by malice or by ignorance confound with *advanced* mathematics, we are interested in *graphs* rather than in shapes, and in *interacting agents* rather than in numbers. Graphs are pictorial symbolic representations for relations existing between interacting agents that are involved in a given operation, or for the behaviour of a given phenomenon, probably with reference to the behaviour of other phenomena. In modern mathematics, pictorial representations are not used to prove anything but only to help understanding the analysis relative to the problem under investigation; the shapes themselves being of minor importance.

In his *Introduction to Arithmetic*, some nineteen centuries ago, Nichomachus of Gerasa [12] presented a conceptualization of *magnitudes* and *multitudes. Things,,* he wrote, *both those properly so called and those that simply have the name, are some of them unified and continuous [...] which are properly and*

peculiary called 'magnitudes'; others are discontinuous, in a side-by-side arrangement [...] which are called 'multitudes'. Multitude, he explained, *starts from a definite root and never ceases increasing; and magnitude, when division beginning with a limited whole is carried on, cannot bring the dividing process to an end, but proceeds therefore to infinity. And since sciences are always sciences of limited things, and never of infinites, it is accordingly evident that a science dealing either with magnitude, per se, or with multitude, per se, could never be formulated, for each of them is limitless in itself, multitude in the direction of the more, and magnitude in the direction of the less. A science, however, would arise to deal with something separated from each of them, with 'quantity', set off from multitude, and 'size', set off from magnitude.* When Nichomachus wrote these words, *things* were conceived with a number-oriented mentality: Multitudes were conceived as the physical incarnation of counting numbers, and magnitudes as the physical incarnation of the *one* and its fractions.

In modern mathematics we are interested in expressions like $A+B$ when A and B are not necessarily numbers! To use such expressions, modern mathematics is deeply concerned with philosophical questions like: *What would be the meaning of* A+B *when* A *and* B *are not numbers*?! As we can feel it, modern mathematics is not a simple modernization of actually existing mathematics. It is rather a new door open to new worlds with which man is unable to deal using his only physical faculties. Sometimes, only mental effort, through symbolic logic analysis, is able to seize the truth of such worlds. *Truth* is used here in a rather relative sense, for in mathematics any "truth" which is true in a given environment of thought might be proven to be false in another thinking environment.

By *thinking environment* we mean the collection of principles and rules upon and through which the truth or the falsity of one's thoughts concerning interacting agents is to be proven. Such thinking environments are called *ALGEBRAS*. So, by *Algebra* we don't make reference anymore to the science but rather to one or another of its environments of thought. With deliberate intension to transgress the reader's traditional ideas, I would like to define *MATHEMATICS*

as THE SCIENCE OF ALGEBRAS! In a such definition, nothing is said about shapes and numbers, and no word is included that makes reference to arithmetic operations or to geometric determinism. A such neutrality is a *sine qua non* postulate if mathematics is to be introduced, as analysis tool, into human and social sciences in general, and into political science in particular.

If our argument removes the interrogation marks relative to *what* mathematics we should deal with, it does not answer the question of *why* mathematics is to be introduced into political science.

TRUTH & MATHEMATICS

If we look backward to trace human and social path through ages, we should reach the conclusion that man's history is a success–failure dichotomy. Indeed, man achieved a great success in his understanding of the physical world around him including his own body. However, this success, that even the inhabitants of Olympus would appreciate and admire, was accompanied by a great failure of man's understanding of himself, that is, of the determinants of his individual and collective behaviour. Even in the cases where man understood these determinants, he was unable to design better behavioural systems. Sometimes, man dreamed of a better world that he created in his mind. But those dreams did never come to existence, simply because man always sculptured his utopias after adult models and not after embryonal ones. Man's achievements in this field always concerned the conceptualization of the ends, not of the chaining causalities leading to them.

Partially by fear and hypocrisy, and partially by missing self–confidence in his reasoning and judgement abilities, man conceived his happiness through models that were, as they actually are, in complete contradiction with the determinants of his behaviour! Consequently, the quasi totality of his behavioural systems were wrongly conceived, and the major part of his truth reference system is nothing but fallacies and inconsistent judgement rules. Some concepts that man invented and considered as control and optimization criteria for his behaviour were so badly conceived that they implicitely led to their own contradictions.

Cleavage between *theory* and *reality* characterized human and social studies for long times. Prejudices that conditioned these studies also conditioned man's understanding of his physical environment. Humanity roved during an irrational era where everything could be respected but not human mind. In applying mathematics to the study of the behaviour of physical phenomena, man overthrew his ideas about nature. He formulated new theorems, made rational thinking to verify them, and predicted the behaviour of natural phenomena which he partially put under his control. Not without sacrifices, rational men forced irrationality to quit physical sciences.

However, if irrationality lost a battle, it did not yet lose the war. It is still queen in the kingdom of human and social sciences. It even became more authoritarian to compensate its losses in the field of physical sciences. Traditions, habits, religious and moral prejudices, national pride, egoism and even stupid interest hierarchical systems, to mention only few, are still influencing our studies in psychology, sociology, economics, history, and all other branches of human and social sciences where political science is not an exception. Irrationality in this family of imperfect sciences has many characteristics:

1. Either terms are not well defined, or they have many different and, sometimes, contradictory definitions with the implications of one definition attributed to the others. Democracy and Socialism are two examples of this imperfection.

2. Primitive propositions, upon the truth of which depends the validity of theorems, are controversial and some of them reveal irrefutable absurdities. It even happens that a primitive proposition, which might be not really primitive, be accepted as a universal original truth that has no need to be proven. A such "truth", which affects all our analyses, might be used in a circular echo process to prove itself!

3. Demonstration methods, through which primitive propositions generate elaborated ones, are either arbitrary or inconsistent. Often, they are both. It even happens that hidden prejudices, which have no legitimate rights to accompany the caravan, cut the route roughly and impose their will on the elaborated propositions.

4. The distinction between what is *universal* and what is *particular* is a mere question of speculation. It frequently happens that a *particular* be affiliated to another *particular* which is arbitrarily taken as its *universal*.

5. The *cause-effect* directionality is circumstantial. It often happens that an effect be considered as the cause of its cause in such a way that conclusions be completely out of logic.

These five aspects of irrational thinking are not exhaustive, but they are enough to show what mathematics can offer to the analysis in human and social sciences in general, and in political science in particular; for in mathematics, none of these imperfections is allowed. In mathematics, terms, even those that sound respectable, should be defined; and no propositions, even those that make us pleasure, are to be accepted unless proven. Proofs themselves are subject to precise and strictly consistent rules. Generalizations are not mathematically tolerated unless through methods that leave nothing to speculation. Finally, the distinction between what is cause and what is effect must be unmistakably clear.

SYSTEMS & ABSTRACTION

As we have mentioned, the main source of imperfection in human and social sciences in general, and in political science in particular, is that scientific neutrality is often difficult if not impossible; for man, who is the thinking subject, is at the same time the object of thought. This raises the more general

problem of *abstraction* which could be better understood if the concept of *system* is introduced. We all employ this term vaguely with little precision and often without any reference to a pre-established definition. When we talk about a certain *political system*, *economic system*, or *social system*, reference is usually made to the political, economic, or social *aspects* of a given society. Let us admit this and let us ask what is really meant by *aspects*. It is a very bad practice that a term be defined using another term which is as vague as the term to be defined.

The concept of *system* is one of the most important and very easy to understand of all the concepts in human and social sciences. A system is a collection of elements that are directly or indirectly related to each others [10 , 20]. Each *element* in the system has a collection of *attributes* that, taken collectively at the system's level, constitute what is called the *state* of the system. Also, every one of the *relations* between the elements of the system has a collection of *behavioural patterns* that, taken together at the system's level, determine the behaviour of the system and, hence, its performance. The *behaviour* of a system is a process in which its elements interact with each others. The *performance* of the system is the output of this interaction. This output might be the conservation of the elements' attributes, as well as the relations between the elements; in which case the system is said to be *retentive*. Another possibility is to change these attributes; which changes often result in a modification of the relations between the elements. In a such case the system is said to be *acquisitive*. The interaction between the elements might take place through the time, in which case the system is said to be *dynamic*. In the case where the interaction takes place implicitly without any reference to time, the system is said to be *static*. The relations through which elements interact with each others might reveal a completely pre-determined pattern, in which case the system is said to be *deterministic*. In the other case where the behavioural pattern is subject to uncertainty, the system is said to be *stochastic*.

Any system could be considered as a subsystem of another system. The rest of the higher order system is called the *environment* of the lower order one. The distinction between the system and its environment might be the object of subjective judgements. However, an objective criterion does exist. In fact, the concept of *environment* is closely related to scientific study. In the real world, this concept of *environment* is meaningless since there is but one system through which both physical and human phenomena take place. Understanding the structure and behaviour of this exhaustive system is impossible if phenomena are not separated from each others. When a certain phenomenon is to be studied, all elements and relations relative to other phenomena should be held constant. The answer to the question of what elements and relations are to be held constant depends on the phenomenon under investigation: the more primitive this phenomenon is, the more are elements and relations that are kept constant; which elements and relations constitute the environment of the system under study.

To carry on any scientific research, we have to begin with separating the appropriate system from its environment. This eliminates many factors that, although any changes in their characteristics will affect the behaviour of the system, are considered as neutrals throughout the study. This is the first step of what is to be referred to by *abstraction process*. The second step is to eliminate those attributes of the system's elements, as well as those behavioural patterns of the system's relations, that are considered as being irrelevant. At the issue of these two steps of the abstraction process, we get a system which is no longer the system of the real world but only a representation of it. More or less faithful to reality, this representation of the real world is called a *model*. For example: if a model is to be obtained for the history of the mankind, the physical world should be considered as a part of the environment. Any changes in the physical laws of nature, although if happened would radically affect this history, are completely discarded. If a such part of the environment is included as a part of the system, then changes of its attributes and its relations would be allowed; but at this case, the study would concern no more the history of the mankind but rather the natural history of the earth.

The abstraction process that we have introduced is far from being complete. What we have made is a *plane concentration of thought*. I call it *plane* because it is done through only two axes; namely, abstraction of the system from its environment, and the abstraction of the most relevant characteristics both of elements' attributes and of relational behavioural patterns. This kind of abstraction is sometimes called *simplification* [20]. A pyramidal concentration of thought is sometimes necessary for that the abstraction process be complete. The plane concentration could be considered as the base of this pyramid. In a higher level of this pyramidal abstraction process all the attributes and behavioural patterns are translated into intellectual forms having the same characteristics of our reduced system. For example: if in our model we had two individual persons related by an additive relation that makes them a pair, then this relation could be translated to the intellectual form 1+1=2 which makes no reference to the fact that they are two men or two women. This level of abstraction might be considered as the base for a higher level that makes no reference to the counting numbers 1 and 2 but to an intellectual structure that has the same properties as the addition of counting numbers.

A such abstraction process eliminates all what we can call the *noise* which disturbs our thinking process. The fact that we are dealing with intellectual structures that make no reference to the specific characteristics of the real world is a *sine qua non* condition of attaining objectivity in our analyses in political science.

Partially because they believed that no speech is acceptable without reasoning, and partially because they believed that no reasoning is possible without speech, ancient Greeks used only one word, *logos*, for both speech and reasoning. If we agree that objective thought is possible only at an abstract level, language to be used at a such level must be abstract as well. Symbolic mathematics was originally invented to keep mathematical argumentation at its abstract level with no reference to the "meaningful" words of our natural languages. Mathematical symbols are meaningless words that would be impossible to use unless through strictly prescribed rules independently from those governing our everyday speech.

No retranslation of these symbols to what they stand for is allowed before that the final conclusion is reached; otherwise, the neutrality obtained by the abstraction process would be violated which is worse than if the abstraction process was not originally engaged.

PYTHAGOREAN EXPERIENCE

In the early dawn of the Human Age, man knew about mathematics what he knew about himself and about the universe around him: absolutely nothing! Progressively, he began to establish a rudimentary conceptual system in which *scale* occupied an eminent place, since the first thing man discovered was the *small* place he occupies in the *big* universe. In this rudimentary conceptual system, the *scale* concept was related to the concepts of *order* and *equilibrium*. These three concepts gave birth to *numbers*, to *equalities* and *inequalities*, and, later on, to *equations*. A whole system of thought is going to be established.

Unfortunately, we know very little about the development of mathematical thinking in ancient oriental civilizations. We are sure that ancient Egyptians and Babylonians had a good, even very good, mathematical knowledge [2 , 9]; but the original manuscripts of these civilizations are lost, presumably for ever. As for the occidental civilization, its mathematical knowledge is said to begin some six hundred years before Christ. At that time, it is supposed that the legendary Thales travelled for Egypt and Babylon, probably as a merchant. Most likely, commerce was not the only objective of Thales; the most important being his curiosity concerning military "engineering". In gathering informations about it, Thales discovered the world of *arithmetic* (the science of numbers), *calculation* (the science of operations between numbers), and *geometry* (the science of shapes). In return to Greece, his compatriots considered him as the inventor of these "arts"; but all his writings, if any, being disappeared, it is difficult to distinguish reality from legend in what concerns his life and his "discoveries". Even his existence is sometimes put in doubt.

Our actual mathematical knowledge begins about 25 centuries ago when an open-minded Greek thinker, Pythagoras of Samos, decided to be a *philosopher* –

a neologism to which he gave the first green–light access to human language[12]. Etymologically speaking, *philosopher* is derived from Greek and literally means *lover of wisdom*. Being *lover* of wisdom means that a philosopher must be conscious of his (or her) defective nature, and consider wisdom as an *ideal* to search, not as an *attribute* to have. William COWPER expressed a meaning near to ours: *Knowledge is proud that he has learn'd so much; Wisdom is humble that he knows no more.* "Wisdom", however, needs to be defined: It is the fuculty to perceive the differences between the *true* and the *false* in order to make correct judgements. To do so, the philosopher must be open–eyed in the intellectual figurative sense of the term. His thinking faculties have to be trained to *objectivity*; that is, to deal with the intrinsic nature of the object of thought rather than with the thinking subject. In the opposite case, when the philosopher believes in some prejudices relative to the object of thought, he (or she) becomes *subjective* and stops to be called *philosopher*.

In his search for the truth, Pythagoras made a journey which chronologically covered his youth, and geographically covered many oriental countries. In the ancient world, Egypt occupied a distinguished rank, and ancient Egyptians had the reputation to be the most intelligent people, and the most authentic of all *philosophers*. During his stay in Egypt, which is not proven, or his contact with Egyptian philosophy, which is attested by Greek writers, Pythagoras discovered a new world, and made contact with men of different intellectual horizons. To this collection of knowledge, to which he had access, Pythagoras gave the name of *mathematikos* which means *scientific subject matters*, derived from *mathema* which means *science* and has its roots in *mathaneo* which means *to learn* or *to acquire*.

Influenced by the Egyptian philosophy, Pythagoras believed that *mathematics* is the only way to approach the truth. Man has no other choice if he wants to make an objective understanding of the universe including himself. There are good reasons, however, to think that Pythagoras had no complete mastery of his thoughts. His mathematical thinking was not very mathematical! The so–called Pythagorean theorem, which states that *the square of the hypotenuse of a right triangle is*

equal to the sum of the squares of its other two sides, is certainly not an original discovery of his own. This theorem was already known to ancient Egyptians and to Babylonians. Even its proof, it was not provided by him but, rather, by his followers. His only contribution was to abuse of this theorem, among others, to establish his religious and political system!

Rather than being intellectually motivated, Pythagoras was fascinated by the idea that everything, including what is out of man's control, could be modeled or represented by idealized shapes and numbers that are subject to study and control. He took the wrong direction. Instead of applying to qualitative aspects of life the rigour of quantitative mathematics, Pythagoras decided to give some qualitative attributes to shapes and numbers; a sore from which we are still suffering today. Mathematics, which is supposed to be the sanctuary of abstraction and objectivity, becomes an alibi for subjective prejudices. The qualitative transfused its defectiveness to the quantitative.

Still, Pythagoras was a great open-minded man. He was in search for the truth, and taught the respect of mathematics and rational thinking. We cann't forget that Pythagoreans themselves have contributed to the abolition of their own religious system when it became in contradiction with truth: A noble devotion to honesty, and fidelity to one's own principles. It was not without vicissitudes. After a long struggle, truth triumphed over fallacy; and the rational deluge carried away Pythagorean thoughts save what was proven to be valid and in complete agreement with the truth.

At least for the occidental world, the advent of Pythagoreanism was a break point between two ages of thought. The first in order is the irrational age during which *how could a "truth" be proven to be true* would appear as being stupid question! For irrational peoples, it is sufficient to believe in something for this something be true! Following their "logic", undefined terms could be used to define others, and the distinction between universal principles and particular realizations is of minor importance; the most important is to *approve* the "truth", not to *prove* it!

Since irrationality does not admit inference and cause–effect implications, irrational peoples had to create their reference authorities. *Someone said that somesecond said... that our venerable ancestor said* was sufficient to establish a "truth" that, in substance, might be false. Since a such process of "thought" inevitably leads to dead ends, irrational peoples had to diversify their reference authorities. *Reverends X and Y studied or learned the speech and deed of our venerable ancestor* was sufficient argument for that X and Y be classified as reference authorities. When you want something to be true, make reference to Reverend X; when you want it to be false, just make reference to Reverend Y; and if you want it to be both, please call Reverend Z, for there are always Z's ready to do the job!! Nobody cares about what is said and why but, rather, about who said it to whom. When an irrational becomes erudite in his art, he cares about more questions like *when* and *where*. To prove a "truth", it was sufficient, it seems, to prove the historicity of an anecdote in which the "truth" is said to be canonically established.

Pythagoras was not an exception. After his death, as it is the case after the death of any great open–minded man, his followers were subdivided into two main intellectual streams: The *mathematics* (derived from the Greek *mathema* which means *science*) and the *acousmatics* (derived from the Greek *akousma* which means *something heard*). The *acousmatics* cared about *what* Pythagoras, the Master, thought, while the *mathematics* were concerned with *how* he thought.

The *acousmatics* ended by an intellectual inertia which was internally passive, and had externally no active power but what the Master has, or supposed to have, said. Pythagoras speech and deed took an aura that no one had the right to approach except to find what makes it more and more influential. *Acousmatics* became more Pythagorean than Pythagoras himself. They are not *Pythagoreans* no more; they are called *Pythagorists.*

Pythagorists, and we still have Pythagorists today, believed that harmony and equilibrium reign over the universe. A fact that we don't deny. They also believed that universal order is numerical, and that numbers could be

classified to benevolent and maleficent numbers! There are also low-level and high-level numbers. *Low-level numbers*, or counting numbers, are created by men. They are used by merchants and, therefore, have no scientific dignity! *High-level numbers*, or transcendental numbers, are created by God to perform universal harmony. Their sense is not perceptible by ordinary peoples and, therefore, never loose their dignity! This is exactly the opposite of what the German mathematician Kronecker said: *God created natural numbers, the rest is the work of man!* [19]. We believe that both are wrong. Numbers are not *created* as dinosaurs to disappear when the environment becomes intolerant and unfavourable to their existence! Numbers with their fantastic world are inherent to ours. If numbers were not what they are, our world would not be what it is. We can imagine the universe without men, without animals, without trees and without mountains. We can imagine it without letters and without words. Indeed, the universe existed before the Bible and before the Koran; but it did never exist without numbers. Without numbers, even God would not be the same. At least He would not be the *one* that we know.

No, we are not converted to pythagoreanism!

Pythagorean thought is not a dim poison but, rather, a poisoned limpidity. The world of numbers with its marvelous architecture is not a mysticism. It is now the core of a whole branche of mathematics: the so-called Theory of Numbers. Transcendental numbers do exist, and the idea that such numbers could be reached through the study of the harmonious "behaviour" of shapes seems to have a good scientific foundation. However, this limpidity does not prove that the drink is potable. Poison drops do exist. The bottle wherefrom one of them comes is labelled *commensurability*.

Commensurability is the state of two magnitudes which have a *common measure* or a *common divisor*. In other words, if A and B are two magnitudes, then each of them is said to be commensurable to the other if there exist two counting numbers m and n such that A and B are equal to mc and nc respectively, where c is the common measure or the common divisor. If two magnitudes are not commensurable to each other, they are called *incommensurable*.

For example, a side of a square is commensurable to any one of the other three sides since, in length, they are to each other what, in magnitude, 1 is to 1. Also, if XY is a line segment, and Z is the midpoint between X and Y , then the line segment XZ is commensurable to XY since XZ is to XY what 1 is to 2 or, in other words, the length of XY is the double of the length of XZ.

One of the ancient known facts in geometry is that if the length of the circumference of a circle is divided by the length of its diameter, the result would be always a constant PI the value of which is slightly greater than 3. It was believed that the circumference of a circle is commensurable to its diameter; that is, PI could be represented by a ratio. This supposed ratio was never found. It was approximated by the ratio 22/7. The ratio 2199/700, however, would give a better result. It was proven that the exact ratio does not exist, but Pythagoras would not believe it! [1 , 21]

The so-called Pythagorean theorem implies that the area of a square Q the side of which is the diagonal of a square S is commensurable to the area of the square S. Indeed, the area of Q is always the double of the area of S. Pythagoras abused of this theorem to establish a religious and political system based on commensurability, that is, the universe is expressible in whole numbers. This system was reduced to nought by the Pythagoreans themselves who proved that if the area of a square is commensurable to the square of its diagonal, the diagonal itself is incommensurable to the square's side. The proof of the existence of incommensurable magnitudes made scandal and started a serious crisis in the history of philosophy. As the Pythagorean Milo of Croton was eaten by wolves while his hands were caught to a tree that he was trying to remove, Pythagoreanism was devoured because it was caught to the tree of commensurable magnitudes, and made absolutely no attention to incommensurables that were eager to prey upon it!

Mathematics was severe in defending its kingdom of Truth. The theorem which by illusion supported Pythagorean philosophy when it was built up, carried in itself what was more than sufficient to pull it down. This led the *mathematics*,

who were influenced by the scientific soul of Pythagoras, to reduce their ritual activities to the minimum, and to have as their first concern not what the Master said but what were the foundations of his sayings. They had the courage and honesty to ask questions about the validity of their Master's ideas. Of course, the *acousmatics*, who were a majority, watched with aversion the evolution of this intellectual stream. It is said that the Pythagorean Hippasus was drawned, simply because he dared make an allusion to the existence of incommensurable magnitudes and, consequently, to the existence of irrational numbers, that is, numbers that could not be expressed in the rational form p/q where p and q are whole numbers. A discovery that turns upside down the very principles of the Pythagorean religious system.

This penumbral age had to come to an end. Day after another, the *acousmatics* were bereaved of their credit. Their obstinate perseverance could not interdict the *mathematics* from proving the validity of their argument and, consequently, the absurdity of the *acousmatic* cause. Under the leadership of Archytas, who was a *mathematic*, Pythagoreans concentrated their thoughts on the rational study of the problems of our existence. While *acousmatics* condamned themselves to a long diaspora, during which they became source of pathetic fallacies, the school of Archytas made contact with the Academy of Plato. Their Pythagorean thoughts made a bridge-head to the future. Because of them, Pythagorean thought had its influence upon mathematical thinking in human and social sciences for a very long time. I even say that the actual mathematical thinking in this family of sciences is still, to some degree, influenced by the Pythagorean thought. Plato is the first great philosopher that was influenced by Pythagoras. In the Republic, he declares that he does agree with the Pythagoreans in that sciences are sisters and that an interdisciplinary study is necessary in order to reach the absolute truth. *But this is a laborious study*, said Socrates to Glaucon, *and therefore we had better go and learn of them* [18].

As Pythagoras, Plato was concerned with mathematics only when it is *pursued in the spirit of a philosopher, and not of a shopkeeper!.* In his Philebus [16], Plato states, in a dialogue between Socrates and Protarchus,

that mathematics is of two kinds, *one of which is popular, and the other philosophical.* This is an echo of the Pythagorean influence upon the Platonic thought. However, the fact that Plato's writings are not lost makes that his ideas be presented in more details and less ambiguity than those of Pythagoras. Popular mathematics deals with *unequal units, as for example, two armies, two oxen, two very large things or two very small things,* while in philosophical or pure mathematics *every unit in ten thousand must be the same as every other unit.* Mathematicians, states Plato in The Republic [18], *although they make use of the visible forms and reason about them, they are thinking not of these, but of the ideals which they resemble; not of the figures which they draw, but of the absolute square and the absolute diameters, and so on... which can only be seen with the eye of the mind.* Pythagoras intellectually survived not because of those who hallowed his speech and deed but, conversely, because of those who doubted, criticized, and proved.

Acousmatics ended by a petrified tree with no leaves and, consequently, no shadow on our age. *Mathematics,* at the contrary, began with living seeds of Pythagoreanism, and these seeds were so vigorous that their dissemination in a different land did not keep them from growing up and giving life to other seeds. The idea of Pythagoras after which mathematics could be applied to understand our existence is still valid; the realization of the idea, of course, was not. At least, it is not valid any more. The knowledge package at Pythagoras' times is not the same after his death. The difference had inevitably its impact on the transmutation of his thoughts, since irrationality has no place in the rational age, then declared. Thomas HOBBES ingeniously ranked the "scientific" erudition of the *acousmatics,* whether they are Pythagorists or not, in these words: *Those men that take their instruction from the authority of books, and not from their own meditation, [are] as much below the condition of ignorant men as men endued with true science are above it. For between true science and erroneous doctrines, ignorance is in the middle* [7]. Of course, since the zero is greater than the negative, and what is still in rest is higher than if it was falling downward.

A post-mortem examination of the Pythagorean experience results in the fact that using good mathematical techniques does not necessarily mean that we employ a good mathematical thinking. Any theorem might be true, but the question is: What theorems to apply to what kind of problems. Applying valid theorems to tackle wrong problems might be source of harm than if the problems were not tackled at all. This problem of matching the theory to the application is of major importance since, in human and social sciences, this is the main source of aberant illusions and, consequently, of irrelevant judgements, inconsistent decisions, and misleading policies. In the Laws of Plato [17], the Athenian stranger said to Cleinias: *I am still more afraid of those who apply themselves to this sort of knowledge, and apply them badly. For entire ignorance is not so terrible or extreme an evil, and is far from being the greatest of all; too much cleverness and too much learning, accompanied with an ill bringing up, are far more fatal.*

It is very important that readers be aware of the danger represented by illusions generated by pseudo-scientific studies not only of numbers but of any other items as well. The fact that such studies make use of numbers, or any other mathematical items or techniques, does not necessarily imply that these studies be affiliated to mathematics. Such studies are comonly called *arithmosophy*. Like rhetoric, this is an art, not a science. It is a natural child of a long irrational tradition that, by fear of the power of Reason and the authority of the scientific search for the truth, made illegitimate contacts with science in order to have some external aspects that agree with the changing environment. As it is the case of lizards, this external transmutation gave irrational thinking a pseudo-appearance of truth which, through atavism, represents a latent danger for the scientific soul. In calling some scientific facts, erroneous conclusions might be formulated to approve the thoughts of the spiritual leaders of irrationalism, the so-called *most learned scholars*. The fact that numbers are related to each others proves only the consistency of the numerical system and not, as some peoples do by illusion, the consistency of human and social behavioural patterns to which numbers are attributed as a measure or as an index criterion.

The main source of danger comes from the fact that invoking pseudo-consistency of numbers and facts touches our defective soul which often falls in the fascination trap. Those who are the masters of this kind of charming art get an appearance of spiritual illumination and intellectual enlightment. The more abusive and impudent the imposture is, the more considerable is the fascination through which the imposture becomes easier to swallow. It is often difficult, and sometimes impossible, for science to disclose the imposture for, in general, men have the defective nature of being apt to be deceived than to admit that they are so. It is their pride which is in balance in face of those who reached the truth. Even in the case where truth is discovered by the same person who was deceived, it is usually very hard for him to admit that he was deceived. It happens to him to live in schizophrenia, that is, to live with the discovered truth without breaking off with fallacies. The most famous example is Blaise PASCAL. This French mathematician and philosopher, who was the inventor of the first calculating machine, did live with mathematical truth side-by-side with his hereditary beliefs [15].

When it is question of faith, mathematics has no answer to give but what the truth reference gives: nothing more, we admit, but nothing less, we do insist. In mathematics, there is no place for *I think* or *I believe*, for faith is subjective while mathematics is the abstract objectivity. Pascal himself, in his scientific writings, does agree with us. In his *On Geometrical Demonstration* he admits that he is far from bringing divine truths under the art of persuasion, *for they are infinitely above nature. God alone can put them into the soul, and in whatever way He pleases... He has willed they should enter into the mind from the heart and not into the heart from the mind* [14].

It is this directionality that makes the difference between what is true science and what is not. A science must take the mind as its starting point. Those "sciences" whose starting point is the heart are not true sciences but only a collection of beliefs. Those who build their studies upon beliefs are not scientists and a fortiori not "the greatest amongst scientists". They are only men of faith. Honest or not, a faith has absolutely no right to be affiliated to science.

EPILOGUE

Under the title of *Loss of Meaning through Intellectual Processes* [4], DAVIS and HERSH wrote: *Abstraction is the source of great benefit and also the source of possible damage. [...] Abstraction is extraction, reduction, simplification, elimination. Such operations must entail some degree of falsification... Whenever anyone writes down an equation that explicitly or implicitly alludes to an individual or a group of individuals, whether this be in economics, sociology, psychology, medicine, politics, demography, or military affairs, the possibility of dehumanization exists. Whenever we use computerization to proceed from formulas and algorithms to policy and actions affecting humans, we stand open to good and to evil on a massive scale. What is not often pointed out is that this dehumanization is intrinsic to the fundamental intellectual processes that are inherent in mathematics.* This point of view is certainly valid. However, it should not be taken as is. Mathematicians never claimed that mathematics is the absolute perfection.

Following Greek mythology [6], someone had the audacity to fall in love with the eternal virgin Artemis. He persued her through the lands of Greece, but she escaped and covered her face with mud, then ordered her maids to do the same. The lover being confused could not distiguish the *true* Artemis from the others. Defeated, he did quit followed by laughs. Another day, a hunter, named Actaeon, surprised Artemis when she was bathing all-naked in a stream passing through the woods. Afraid that he might pride himself before his friends that she took off her clothes in his presence, Artemis converted the poor Actaeon into a stag, and he was devoured by his own dogs.

When she asked her father an eternal virginity, Artemis was decided not to show her naked body to any *hunter*. Naked truth is not available for mortals who are not strong enough to support its consequences. Though it does not give a superhuman perfection to capture the naked truth, mathematics has all the perfection men are capable of [14]. What is suggested is that both natural

and human orders be studied through the same mathematical thinking approach, which proved itself in the study of the former. Ancient Greeks suggested this unicity in one of their beautiful tales. It is said that Niobe, queen of Thebes, had seven sons and seven daughters of whom she was eccessively proud. One day, Niobe injured Leto and her children: Artemis and Apollo. *I am the queen of Thebes,* she said, *while Leto was nothing but a miserable woman with no refuge. She gave birth to only two children while I have many.* Artemis and Apollo avenged their mother and killed all the sons and daughters of Niobe. *Together,* the two gods, offspring of the original truth, had the victory over fallacies in their multitude. This *suggestion* of approach synchronization is not, of course, a proof for anything.

Davis and Hersh [4] *suggest,* they too, that *the world is the sepulchre, not only of famous men, but also of dead thoughts, issues, beliefs, customs, rituals, actions... The Divine Right of Kings died under the guillotine with Louis of France. The center of our planetary system has been moved from the earth to the sun. Beliefs systems are pulverized, and meaning drains from act and object, even as the leaf falls from the tree and returns to dust before our eyes... Losses of meaning occur when intellectual ideals come into conflict. Such conflicts are most often centered around one question: "which forms of knowledge are best or most important?". In more virulent conflicts, the question becomes: "which forms of knowledge are true and good, and which are false and evil?".* Exactly two centuries before Davis and Hersh, Immanuel Kant wrote: *Nothing can possibly be conceived in the world, or even out of it, which can be called good, without qualification, except a good will* [8]. Again, this does not *prove* anything; it only *suggests.*

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